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Resonant frequency temperature coefficients for the piezoelectric resonators working in various vibration modes

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Abstract Resonant frequency temperature coefficient is dependent on material properties, resonator dimensions and vibration mode. It could be effectively tuned by the resonator dimensions or by the domain structure in ferroelectric crystals. Optimum dimensions for the zero temperature coefficient resonator are calculated for ring radial vibration mode as a function of resonator dimensions (r_2/r_1=3.8 for hard PZT ceramics, APC841 type). There are similar results of the temperature coefficient calculations for PZT ceramics and crystal resonators. The temperature coefficient is generally smaller for higher overtones of resonant mode.

Keywords Temperature coefficients · PZT ceramics · Piezoelectric resonators

1 Introduction

Resonant frequency and its temperature stability is one of the most important parameters for any piezoelectric resonator. The resonant frequency temperature coefficient is dependent on the material properties of resonator material as well as on the resonator dimensions and vibration mode. Resonant frequency itself and its temperature dependence could be tuned by choosing proper crystallographic cut or proper overtone frequency for single-crystals such as quartz (see e.g., [1–2]), by choosing proper dimensions or vibration mode in ceramic resonators or by the domain engineering [3] in ferroelectric crystals. Resonator impedance (admittance) for various vibration modes can be deduced from the equation of motion and piezoelectric linear equations of state under proper approximation given by the material symmetry, sample shape and its dimension aspect ratio [4–7]. However data on temperature dependences for electromechanical properties (including temperature coefficients) are extremely rare in the literature for piezoelectric ceramics—e.g., [8–9].

This paper is focused on the systematic theoretical analysis of the temperature coefficients for piezoelectric ceramic resonators for the length-extensional, thickness-extensional, thickness-shear and radial vibration modes for the resonant frequency of homogeneously as well as non-homogeneously poled ceramic resonators. The same temperature properties are calculated also for $LiTaO_3$, $LiNbO_3$ and $Pb_5Ge_3O_{11}$ domain engineered crystal resonators working in radial vibration mode for comparison.

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2 Results

The linear temperature coefficient for quantity x at the temperature Θ_0 is defined as [1]

$$\mathrm{TK}(x) = \frac{1}{x}\left(\frac{\partial x}{\partial \Theta}\right)_{\Theta_0} \tag{1}$$

Various resonator shapes are shown in Fig. 1. For homogeneously poled resonators, resonance condition (obtained for resonators impedance equal to zero) is

$$\eta_r = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots \tag{2a}$$

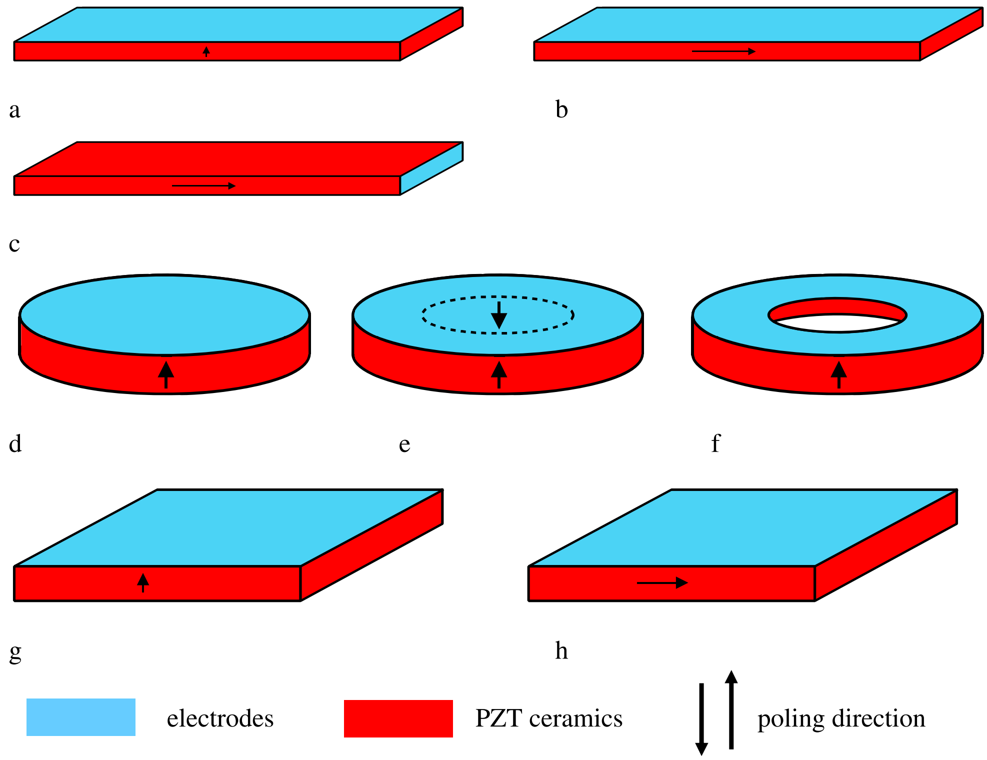

Fig. 1 Resonator shapes. **a** transversal vibrations of thin bar, **b** thickness shear vibrations of thin bar, **c** longitudinal vibrations of thin bar, **d** radial vibrations of thin disc, **e** radial vibrations of thin disc non-homogeneously poled, **f** radial vibrations of thin ring, **g** thickness extensional vibrations of thin plate, **h** thickness shear vibrations of thin plate

where

$$\eta_r = \pi f_r l \sqrt{\rho s_{11}^E} \tag{2b}$$

and

$$\mathrm{TK}(\eta_r) = 0 \tag{2c}$$

for length-extensional vibrations of transversally poled thin bar (Fig. 1a). It is similarly

$$\tan(\eta_r) = \frac{1}{k_{33}^2}\eta_r \tag{3a}$$

$$\eta_r = \pi f_r l \sqrt{\rho s_{33}^D} \tag{3b}$$

$$\frac{\mathrm{TK}(\eta_r)}{\mathrm{TK}(k_{33})} = \frac{2k_{33}^2}{k_{33}^2\left(1-k_{33}^2\right)-\eta_r^2} \tag{3c}$$

for length-extensional vibrations of longitudinally poled thin bar (Fig. 1c),

$$\tan(\eta_r) = \frac{1}{k_{15}^2}\eta_r \tag{4a}$$

$$\eta_r = \pi f_r t \sqrt{\rho s_{44}^D} \tag{4b}$$

$$\frac{\mathrm{TK}(\eta_r)}{\mathrm{TK}(k_{15})} = \frac{2k_{15}^2}{k_{15}^2\left(1-k_{15}^2\right)-\eta_r^2} \tag{4c}$$

Table 1 Temperature coefficients for the first three overtones for ceramic resonators (APC841).

Resonator	Frequency constant η_r [1]			Frequency constant temperature coefficient ratio $TK(\eta_r)/TK(k_{i\alpha})$[1]			
	1st	2nd	3rd	1st	2nd	3rd	Equation no.
Fig. 1c	1.24319	4.62122	7.79987	−0.649	−0.040	−0.014	3c
Fig. 1b, h	1.28996	4.63223	7.80635	−0.520	−0.035	−0.012	4c
Fig. 1g	1.46752	4.67990	7.83457	−0.150	−0.014	−0.005	5c
Fig. 1d	2.02359	5.38169	8.56713	0.082	0.009	0.004	6c

Dimensionless ratios of the temperature coefficient for frequency constants are calculated according to equations whose numbers are listed in the last column.

for thickness-shear vibrations of thin plate or bar (Fig. 1b or h),

$$\tan(\eta_r) = \frac{1}{k_t^2}\eta_r \tag{5a}$$

$$\eta_r = \pi f_r t\sqrt{\frac{\rho}{c_{33}^D}} \tag{5b}$$

$$\frac{\mathrm{TK}(\eta_r)}{\mathrm{TK}(k_t)} = \frac{2k_t^2}{k_t^2\left(1-k_t^2\right)-\eta_r^2} \tag{5c}$$

for thickness-extensional vibrations of thin plate (Fig. 1g),

$$\frac{\eta_r J_0(\eta_r)}{J_1(\eta_r)} = 1-\sigma^p \tag{6a}$$

$$\eta_r = 2\pi f_r r\sqrt{\frac{\rho}{c_{11}^P}} \tag{6b}$$

$$\frac{\mathrm{TK}(\eta_r)}{\mathrm{TK}(\sigma^P)} = \frac{-\sigma^P}{1-(\sigma^P)^2-\eta_r^2} \tag{6c}$$

for radial vibrations of thin disc (Fig. 1d),

$$\frac{\left[1-(1-\sigma^p)Y\left(\frac{r_2}{r_1}\eta_r\right)\right][1-(1-\sigma^p)J(\eta_r)]}{\left[1-(1-\sigma^p)J\left(\frac{r_2}{r_1}\eta_r\right)\right][1-(1-\sigma^p)Y(\eta_r)]} = \frac{Y_0(\eta_r)}{J_0(\eta_r)}\frac{J_0\left(\frac{r_2}{r_1}\eta_r\right)}{Y_0\left(\frac{r_2}{r_1}\eta_r\right)} \tag{7a}$$

where η_r is the same as in Eq. 6b, for radial vibrations of thin ring (Fig. 1f) and

$$Y_0(\eta_r)J_0\left(\frac{r_2}{r_1}\eta_r\right)[Y(\eta_r)-J(\eta_r)]\left[1-\left(1-\sigma^P\right)J\left(\frac{r_2}{r_1}\eta_r\right)\right] = 0 \tag{8a}$$

where η_r is the same as in Eq. 6b, for radial vibrations of thin disc poled in thickness direction inside the radius r_1 and in antiparallel thickness direction between radii r_1 and r_2. In the equations above, ρ means density of the resonator, s_{11}^E, s_{33}^D, s_{44}^D are elastic compliances, c_{33}^D is elastic modulus, k_{33}, k_t, k_{15} are electromechanical coupling factors for longitudinal, thickness and shear vibrations, l, t, r are length, thickness and radius of the resonator and f_r is the resonant frequency. Planar elastic moduli are defined as [4]

$$c_{11}^P = c_{11}^E - \frac{\left(c_{13}^E\right)^2}{c_{33}^E}, c_{12}^P = c_{12}^E - \frac{\left(c_{13}^E\right)^2}{c_{33}^E}, \sigma^P = \frac{c_{12}^P}{c_{11}^P} \tag{9}$$

and modified Bessel functions (expressed through Bessel functions of the first kind and zero and first order) as

$$J(x) = \frac{J_1(x)}{xJ_0(x)}, \quad Y(x) = \frac{Y_1(x)}{xY_0(x)} \tag{10}$$

Temperature coefficients are not listed here because of long algebraic expressions obtained generally by temperature derivative of the resonance conditions (see Eqs. 7a or 8a).

If the temperature coefficients for elastic moduli, compliances, electromechanical coupling factors, Poisson's ratio and thermal expansion are known for particular material, also the temperature coefficient for the resonant frequency could be calculated. For example, combining Eqs. 1, 2b and 2c, temperature coefficient of the resonant frequency is expressed as

$$\mathrm{TK}(f_r) = \mathrm{TK}(\eta_r) + \frac{1}{2}\alpha_{33} - \frac{1}{2}\mathrm{TK}\left(s_{11}^E\right) \tag{11}$$

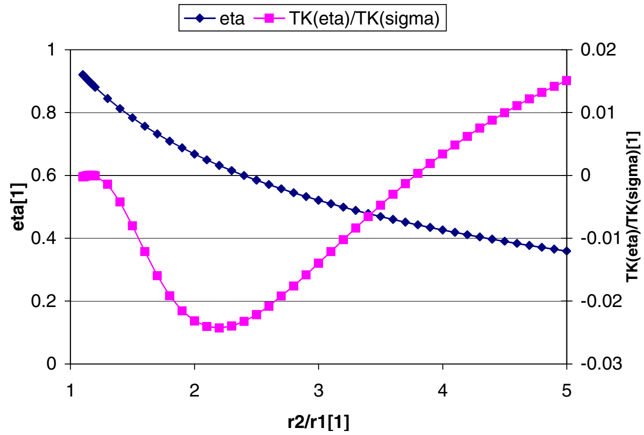

Fig. 2 Dimensionless temperature coefficients ratio $TK(\eta_r)/TK(\sigma^P)$ for the frequency constant η_r (see definition in Eq. 6b) of ring ceramic resonator (see Fig. 1f) as a function of the radii ratio r_2/r_1 (inner radius r_1, outer radius r_2). Calculated for hard PZT (APC841, APC International, Ltd., Mackeyville, PA, USA)

Table 2 Material properties of PZT ceramics (APC841) and crystals used in calculations.

Material properties of PZT ceramics and crystals				
PZT ceramics (APC 841)				
	σ^P [1]	k_{33} [1]	k_t [1]	k_{15} [1]
	0.26	0.65	0.39	0.61
Single-crystals				
	Crystal	$LiNbO_3$	$LiTaO_3$	$Pb_5Ge_3O_{11}$
	σ^P [1]	0.167	0.099	0.052

Table 3 Dimensionless temperature coefficients ratio $TK(\eta_r)/TK(\sigma^P)$ for the first overtone for ceramic and domain engineered crystal resonators.

Dimensionless temperature coefficients ratio	
PZT (APC841)	−0.0832
$LiNbO_3$	−0.0569
$LiTaO_3$	−0.0354
$Pb_5Ge_3O_{11}$	−0.0193

Resonator's geometry is non-homogeneously poled thin disc (see Fig. 1e); r_2/r_1=2; fundamental frequency for planar mode.

where α_{33} is thermal expansion coefficient in the direction of ceramics polarization. Temperature coefficients for frequency constants η_r calculated for homogeneously poled ceramic resonators for the first three overtones are listed in Table 1. (resonator geometries in Figs. 1b, c, d, g, h). We can clearly see decrease of the temperature coefficients for higher overtones for all resonator modes. Material coefficients used for calculations are listed in Table 2. It is interesting to notice, that the temperature coefficient of frequency constant is dependent only on electromechanical coupling factor and dimensionless frequency constant η_r itself (except of planar vibrations of thin disc, where it depends on Poisson's ratio σ^P).

Resonance condition for all studied homogeneously poled resonators does not depend on geometrical dimensions of the resonator. The same is valid also for the temperature coefficient of the frequency constant. Therefore the temperature coefficient of the frequency constant for homogeneously poled resonator might not be tuned by resonator's dimension. Other situation happens for the ring resonator (two different radii structure) or domain engineered disc resonator. In these cases, the resonance condition is dependent on the inner/outer radii ratio and therefore also calculated temperature coefficients follow this dependence. It opens new possibility how to tune temperature coefficient for piezoelectric resonator by its dimensions. The temperature coefficient of frequency constant as a function of the resonator geometry is plotted in Fig. 2 for ring ceramic resonator. Temperature coefficient for frequency constant is dependent on the radii ratio for radial vibrations of thin ring. Optimum ratio for the zero temperature coefficient resonator is $r_2/r_1=3.8$ for studied PZT material. If the temperature coefficients for c_{11}^P, ρ and thermal expansion α_{11} are known, temperature coefficient for the resonant frequency f_r can be calculated. Unfortunately, data on temperature coefficients of material properties are extremely rare in the literature, especially for ceramic materials.

Employing ferroelectric domains in non-homogeneously poled structure is another possibility how to tune temperature coefficient in ferroelectric materials. Such method is widely used in optical structures for second harmonic generation these days, but its application in piezoelectric resonators is just at the beginning. Values of the temperature coefficients similar in magnitude are demonstrated for PZT ceramics and for domain engineered single crystals for non-homogeneously poled domain engineered disc in Table 3, even though the planar Poisson ratios σ^P are very different for these materials.

3 Conclusions

Formulae for the temperature coefficients of the frequency constant were derived for various resonator shapes and resonant modes for homogeneously as well as non-homogeneously poled resonators. Temperature coefficients for frequency constant decrease for higher overtones and are similar in magnitude for ceramics and crystals. For thin ring resonator, temperature coefficient could be further tuned by the resonator's geometry.

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